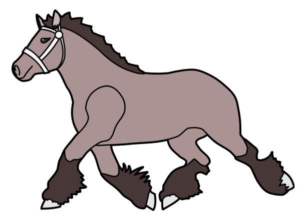
Problem of the Week
Problem C and Solution
And On This Farm

Problem

"Old MacDonald had a farm, E-I-E-I-O", says the old children's song. But Old MacDonald did have a farm! And on that farm he had some horses, cows, pigs and 69 water troughs for the animals to drink from. Only horses drank from the horse troughs, exactly two horses for each trough. Only cows drank from cow troughs, exactly three cows per trough. And only pigs drank from pig troughs, exactly eight pigs per trough. Old MacDonald's farm has the same number of cows, horses and pigs.

How many animals does Old MacDonald have on his farm?



Solution

Solution 1

Let n represent the number of each type of animal.

Since there are two horses for every horse trough, then n must be divisible by 2. Since there are three cows for every cow trough, then n must be divisible by 3. Since there are eight pigs for every pig trough, then n must be divisible by 8. Therefore, n must be divisible by 2, 3 and 8. The smallest number divisible by 2, 3, and 8 is 24. (This number is called the *lowest common multiple* or LCM, for short.)

If there are 24 of each kind of animal, there would be $24 \div 2 = 12$ troughs for horses, $24 \div 3 = 8$ troughs for cows and $24 \div 8 = 3$ troughs for pigs. This would require a total of $12 + 8 + 3 = 23$ troughs. Since there are 69 troughs and $69 \div 23 = 3$, we require 3 times more of each type of animal. That is, there would be $24 \times 3 = 72$ of each type of animal. The total number of animals is $72 + 72 + 72$ or 216.

We can check the correctness of this solution. Since there are two horses for every horse trough, there are $72 \div 2 = 36$ horse troughs. Since there are three cows for every cow trough, there are $72 \div 3 = 24$ cow troughs. Since there are eight pigs for every pig trough, there are $72 \div 8 = 9$ pig troughs. The total number of troughs is $36 + 24 + 9 = 69$, as expected.

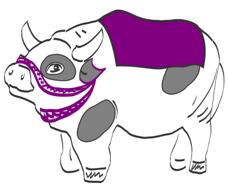
Solution 2

In this solution, algebra and equation solving will be used to solve the problem.

Let n represent the number of each type of animal.

Since there are n horses and there are two horses for every horse trough, then there would be $\frac{n}{2}$ troughs for horses. Since there are n cows and there are three cows for every cow trough, then there would be $\frac{n}{3}$ troughs for cows. Since there are n pigs and there are eight pigs for every pig trough, then there would be $\frac{n}{8}$ troughs for pigs. Since there are 69 troughs in total,

$$\begin{aligned}
\frac{n}{2}+\frac{n}{3}+\frac{n}{8} &= 69 && \\
\frac{12n}{24}+\frac{8n}{24}+\frac{3n}{24} &= 69 && \text{common denominator 24} \\
\frac{23n}{24} &= 69 && \text{simplify the fractions} \\
23n &= 24\times 69 && \text{multiply both sides by 24} \\
23n &= 1656 && \text{simplify} \\
n &= \frac{1656}{23} && \text{divide both sides by 23} \\
n &= 72 &&
\end{aligned}$$



There are 72 of each type of animal, a total of 216 animals.

Solution 3

In this solution, ratios will be used to solve the problem.

Let n represent the number of each type of animal. The ratio of the number of troughs required for the pigs to the number of troughs required for the cows is

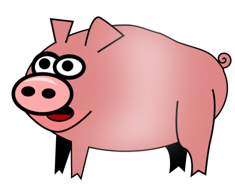
$$\frac{n}{8}:\frac{n}{3}=\frac{3n}{24}:\frac{8n}{24}=3n:8n=3:8.$$

Similarly, the ratio of the number of troughs required for the cows to the number of troughs required for the horses is $2:3=8:12$. So the ratio of the number of troughs required for the pigs to the number required for the cows to the number required for the horses is $3:8:12$.

Let the number of troughs required for the pigs be $3k$, for the cows be $8k$ and for the horses be $12k$, for some positive integer value of k.

Since the total number of troughs required is 69, then

$$\begin{aligned}
3k+8k+12k &= 69 \\
23k &= 69 \\
k &= 3
\end{aligned}$$



The number of troughs required for the pigs is $3k=9$. There are 8 pigs at each trough. There are a total of $9\times 8=72$ pigs. Since there are the same number of each animal, there are also 72 cows and 72 horses. There are a total of $72+72+72=216$ animals on Old MacDonald's farm.